Nonlinear Quasi-Bessel Beam Generation Based on the Time-domain Digital-Coding Metasurface

FANG Zuqi  CHENG Qiang  CUI Tiejun*

(State Key Laboratory of Millimeter Waves, Southeast University, Nanjing 21189, China)

Abstract: A quasi-Bessel beam is a type of nondiffracted beam commonly used in microwave and optical fields. Although numerous methods have been proposed for quasi-Bessel beam generation, they are valid only in linear systems, indicating that the generation of nonlinear quasi-Bessel beams remains a major challenge. Thus, we propose a new approach to produce quasi-Bessel beams at high-order harmonics based on the time-domain digital-coding metasurface, which is utilized to achieve accurate control of the phase profile at the nonlinear frequencies via proper coding strategies. The effect of phase discretization is also analyzed in detail. The simulation results confirm the validity of the proposed method, which provides a new approach for nonlinear beam manipulation.

Key words: Time-domain digital coding metasurface; Bessel beam; Nonlinear modulation

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1 Introduction

Bessel beam is a kind of non-diffracted beam (or localized wave). Due to the collimation and non-diffraction, Bessel beams can concentrate the energy of electromagnetic waves in a small space. It is widely used in many applications such as wireless power transmissions, secure communications, laser micromachining, and so on.

In 1987, Durnin proposed the concept of Bessel beam[1], which derives from a set of solutions of Helmholtz equation in cylindrical coordinates. Theoretically, the ideal Bessel beam is unrealizable since it requires infinitely large energy. However, the truncated Bessel beams, or pseudo-Bessel beams were discovered to remain non-diffracted over a long distance[1–4]. In optics, the
pseudo-Bessel beam could be produced through circular slit and lens\cite{1}, or by the axicon\cite{4}. In General, the latter has advantages of high efficiency and convenience, but usually occupies a large space for the beam generation. To overcome this limitation, some new approaches have been developed such as the the Fabry-Perot resonator and reflector.

In contrast, the axicon is no longer suitable for Quasi-Bessel beam generation in microwave region, due to the large size of the overall system, hindering the system integration. As a result, metasurfaces are developed to control the wave front\cite{5,6} to solve those problem. Recently, the antenna array has been proposed to generate Quasi-Bessel beams with low profile\cite{2}, but complex feeding networks were required to achieve this goal, and the system bandwidth was highly limited by the antenna performance. In the past few years, a series of studies have been focused on the Quasi-Bessel beam generation based on the metamaterial lens\cite{7,8}. The broadband phase profile can be easily implemented by optimizing the meta-atoms, thereby the metamaterial lens is especially advantageous for smaller thickness, simpler structure and wider bandwidth compared to other approaches.

In some application scenarios, especially in wireless communications, the Simultaneous Power and Data Transmission (SPDT) is highly desired, as it can extend the operation time and increase the communication distance of portable devices. To further enhance the channel capacity of the whole system and increase spectral efficiency, a number of subchannels at different carrier frequencies are introduced to transmit orthogonal modulated signals. A possible recipe to realize SPDT is to use nonlinear Quasi-Bessel beams at multiple harmonics. The beams can be focused at different user ends, and achieve efficient energy transfer while transmitting and receiving information between the base-station and users through numerical sub-channels.

However, the nonlinear responses of traditional metamaterials are very weak, making it hard to produce the Quasi-Bessel beams at multiple frequencies. To tackle this problem, time-domain digital coding metasurfaces have been proposed for the nonlinear manipulations of electromagnetic waves during the wave-matter interactions\cite{9–17}. By applying the periodic modulations on the reflectivity of the metasurfaces, it is possible to control the harmonic properties accurately, thus paving a new avenue for nonlinear wave control with ultra-high efficiency.

In this paper, we report the generation of nonlinear Quasi-Bessel beam with the time domain digital coding metasurface. By carefully choosing the coding strategy, the harmonic phase profiles of the metasurface can be obtained to synthesize the desired Quasi-Bessel beams, with high energy conversion efficiency from the fundamental frequency to the specified harmonic. The theory is validated by the numerical simulations.

2 Design of Metasurface for Non-linear Quasi-Bessel Beam Generation

From the classical electromagnetic theory, the electromagnetic fields in free space are expressed by homogeneous wave equation as

\[ \left( \nabla^2 - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \right) \psi (\vec{r}, t) = 0 \] (1)

where \( \vec{r} \) is the position vector, \( \psi \) represents the electromagnetic field, \( t \) is time and \( v \) is the phase velocity.

Some solutions of the wave equation allow the localization of electromagnetic waves\cite{1}, with the energy concentrated within a specific region. The Bessel beam derives from such a solution of localized electromagnetic wave in cylindrical coordinate system\cite{3}. The electromagnetic field of Bessel beam can be expressed as

\[ \psi (\rho, \phi, z, t) = A_0 J_0 (k_\rho \rho) e^{j(\beta z - \omega t)} \] (2)

where \( J_0 () \) is the Bessel function of the zero order, \( k_\rho \) and \( \beta \) are the radial and longitudinal components of the wave vector respectively, which are related by

\[ \beta^2 + k_\rho^2 = k_0^2 \] (3)

The angle between \( k_\rho \) and \( \beta \) can be expressed as

\[ \delta = \arctan \left( \frac{k_\rho}{\beta} \right) \] (4)
It describes a cone with an opening angle of 2\(\delta\)\(^{[1]}\).

As shown in Fig. 1, assume a monochromatic wave incident toward the metasurface at normal direction, in order to get a nonlinear Bessel beam at the frequency \(f_1\), Eq. (3) needs to be re-written as
\[
\beta^2 + k_{\rho}^2 = k_1^2 \quad (5)
\]

Fig. 1 shows the angle \(\delta\) between the surface normal and the plane wave vector in a cut-plane. This angle determines the shape of the truncated Bessel beam in free space. From Ref. [2], the phase distribution of the metasurface that is used to generate the zero-order Bessel beam at \(f_1\) can be expressed as\(^{[3]}\)
\[
\varphi(\vec{r}) = k_1 \|\vec{r}\| \sin \delta \quad (6)
\]
which is illustrated in Fig. 2. Here \(\vec{r}\) is the displacement vector between the element and the metasurface center, and the \(k_1\) is the wave number at the harmonic frequency. The phase distribution is symmetric with respect to the center, which is consistent to the symmetry of the generated beam.

The relationships between \(\beta\), \(k_\rho\) and \(k_1\) are described by Eq. (5). According to Eqs. (2–5), the Bessel beam is composed of an infinite number of plane waves. Moreover, the Fig. 1 shows a section of a nonlinear Quasi-Bessel beam. Quasi-Bessel beam can be considered as a superposition of plane waves, whose radial components of the wave vector is equal to \(k_\rho\).

Two crucial problems remain to be solved to produce a nonlinear Quasi-Bessel beam. The first is how to generate the desired harmonic with high efficiency. The other is how to acquire the accurate harmonic phase profile as described in Eq. (6). As discussed in Ref. [18], for a metasurface with time varying reflectivity, the reflected electric field \(E_r(t)\) can be expressed as\(^{[18]}\)
\[
E_r(t) = \Gamma(t) E_i(t) = \Gamma(t) e^{i2\pi f_0 t} \quad (7)
\]
where \(E_i(t)\) and \(E_r(t)\) are the incident and reflected electric fields respectively, and \(f_0\) is the frequency of the incident wave. \(\Gamma(t)\) is the reflectivity of metasurface. When the incident wave is a monochromatic wave with \(E_i(t) = e^{i2\pi f_0 t}\), from the Fourier transform the reflected spectrum of \(E_r(f)\) can be expressed as
\[
E_r(f) = \Gamma(f) * E_i(f_0) = \Gamma(f - f_0) \quad (8)
\]

If the reflectivity of metasurface signal \(\Gamma(t)\) is designed as\(^{[18]}\)
\[
\Gamma(t) = e^{i2\pi \frac{(t-nT)}{T}}, t \in (nt, (n+1)t) \quad (9)
\]

Fig. 3(a) shows the phase of the reflectivity \(\Gamma(t)\). The time is normalized by the modulation

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**Fig. 1** Schematic of the nonlinear Quasi-Bessel beam generation based on the metasurface

**Fig. 2** Phase distribution of the metasurface in order to get a nonlinear zeroth-order Quasi-Bessel beam

**Fig. 3** Dependence of the metasurface reflectivity on normalized time (a) and generation of the first order harmonic at the incidence of the monochromatic wave with the carrier frequency of \(f_0\) (b)
period $T$. It is clear that the phase needs to be changed linearly within each period. It has been revealed that the reflectivity shown in Fig. 3 can be used to convert the incident wave into the specified harmonics almost completely because it can effectively suppress higher harmonics\[18\].

Although from Eq. (9) we are able to obtain a nonlinear harmonic with high efficiency. However, it is hard to control the harmonic phase of the meta-atoms as needed. This problem can be resolved by introducing a small time delay $\tau$ in $E_r(t)$,

$$E_r(t) = \Gamma(t - \tau) E_i(t) = \Gamma(t - \tau) e^{2\pi f_0 t} \quad (10)$$

The reflected wave in the frequency domain can be expressed as

$$E_r(f) = \Gamma(f - f_0) \quad (11)$$

where the frequency of the incident wave is $f$.

The phase shift of the first harmonics caused by time delay is $-2\pi f \tau$\[18\]. Fig. 4 shows the dependence of the first harmonic phase ($f = f_0$) on the time delay $\tau$, where is normalized to the period $T$. It shows that a full phase range can be achieved by selecting proper $\tau$, leading to the independent control of the harmonic phase required for the generation of Quasi-Bessel beam at nonlinear harmonic frequencies.

$$2\pi \quad 3\pi/2 \quad \pi \quad \pi/2$$

$$0 \quad 50 \quad 100$$

Fig. 4 Dependence of the first harmonic phase on the delay time (Time delay quantization effects are taken into account)

Based on the aforementioned modulation strategy, we are able to design a time domain digital coding metasurface for nonlinear Quasi-Bessel beam generation. Here the metasurface is designed to operate at 12 GHz. The modulation frequency is set to be 100 MHz which can be generated by FPGA. Therefore the first order harmonic frequency is 12.1 GHz, and the conversion efficiency from the incident wave to the first order harmonic is 100% in theory.

For the digital coding metasurface, the reflection phase is usually discrete instead of continuous when the PIN diodes are used in the element design. The phase discretation leads to small degradation of the conversion efficiency as revealed in[18], which means more harmonics emerges in this case. This is clearly illustrated in Fig. 5.

But the independent amplitude/phase is not affected. In addition, the phase profile of the whole metasurface is also discretized, and its influence on the beam generation needs to be further evaluated.

3 Numerical Results

In general, the Quasi-Bessel beam from the metasurface could be decomposed into a series of plane wave, making it possible to calculate the beam field distributions through numerical simulations. Here the commerical software package MATLAB is used to simulate the generated nonlinear Quasi-Bessel beam through the nonlinear modulation of the metasurface. The metasurface is made of 41×41 elements in all, which are arranged in a rectangular pattern. The period of the element is 10 mm, and the cone angle of Quasi-Bessel beam $\delta = 15^\circ$.

From the linear property of Fourier transform, we have

$$F \left( \sum_{i}^N \Gamma_i(t) e^{-jkr_i} \right) = \sum_{i}^N F(\Gamma_i(t) e^{-jkr_i}) \quad (12)$$

where $F$ is the Fourier transform operator, $\Gamma_i$ is the transient reflectivity on the $i$-th meta-atom. $k$ is the wave number in free space. $r_i$ is the distance from the $i$-th meta-atom to the observation point. From Eq. (12), we can calculate the amplitude and phase of the first harmonic for each
element, and then add up all the coefficients to obtain the amplitude and phase of the first harmonic generated from the whole metasurface, as it can help reduce the calculation complexity in the simulation.

The electric fields in front of the metasurface can be obtained after removing the time variable as follows:

\[ E(\mathbf{r}) = \sum_{i} \sum_{j} A_{ij} e^{(k ||\mathbf{r}_{ij}|| + \Delta p_{ij})} \frac{1}{4\pi ||\mathbf{r}_{ij}||^2} \tag{13} \]

where \( ||\mathbf{r}_{ij}|| \) is the distance between the meta-atom and the observed point, \( A_{ij} \) is the reflection amplitude of the \( i \)-th element, \( \Delta p \) is the phase difference caused by the small delay \( \tau \). As indicated in Ref. [18], we prefer to use discrete phases instead of continuous phases to simplify the design of the digital coding metasurface. Here we use 2 bit coding metasurface with four phase states 0, 90, 180 and 270° respectively. However, the phase discretization also gives rise to the wavefront distortion for the Quasi-Bessel beam. So the influence needs to be further evaluated through simulation.

Fig. 6(a), Fig. 6(b) show the required continuous and discrete harmonic phase profile of the metasurface for nonlinear Quasi-Bessel beam generation.

According to Eq. (13), the electric fields in the space can be obtained by discretizing the space and calculating the field at each point. By executing such a numerical simulation, some images of the electric field are attained.

Figs. 7(a) and (b) demonstrate the simulated electric field distributions of the first harmonic at a cut-plane perpendicular to the metasurface, corresponding to the cases of continuous and discrete phase respectively. By comparing the two figures, we can find that the mainlobes and sidelobes are slightly disturbed by the phase quantization errors, and the focusing ability is kept well as expected.

Fig. 8 shows another snapshot of the simulation electric field distributions for the first order harmonic at 12.1 GHz, when the coding metasurface with discrete phase states are employed. It can be seen that the beam is collimated at the center with the intensity larger than that in the rest of the regions. Due to the finite period of the meta-atom, the sidelobes are also obvious around the beam center. However, the influence is quite limited according to the simulation results.
4 Conclusion

In this paper, we realize the generation of an nonlinear Quasi-Bessel beam by a time-domain digital coding metasurface. To achieve the required phase profile of the metasurface in order to get a Quasi-Bessel beam at the first harmonic frequency, a periodic reflection coefficient with triangular phase distributions is considered to get a high conversion efficiency from fundamental to first order harmonic, and a time delay is used to obtain the harmonic phase for each element. Numerical simulations reveal that the nonlinear Quasi-Bessel beam can be generated as expected, and the discrete phase states introduced by the digital coding metasurface have limited impact on the beam properties, which agree well with theoretical predictions.

References


FANG Zuqi (1995–) is a doctoral candidate at Southeast University. His main research interests are passive phased arrays, smart antennas and metamaterials.
E-mail: 230209030@seu.edu.cn

CHENG Qiang (1979–) received the B.S. and M.S. degrees from the Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 2001 and 2004, respectively, and the Ph.D. degree from Southeast University, Nanjing, in 2008. In 2008, he joined the State Key Laboratory of Millimeter Waves, Southeast University, where he was involved in the development of metamaterials and metadevices. He is currently a Full Professor with the School of Information Science and Engineering, Southeast University. He leads a group of Ph.D. students and master’s students in the areas of metamaterials, tunable microwaves circuits, microwave imaging, and terahertz systems. He has authored or co-authored more than 100 publications, with citation over 2000 times.
E-mail: qiangcheng@seu.edu.cn

CUI Tiejun (1965–) is the academician of Chinese Academy of Sciences and the Chief Professor of Southeast University, Nanjing, China. He authored or co-authored two books and published over 500 peer-review journal papers, which have been cited by more than 35000 times (H-index 93, Google Scholar). He proposed the concepts of digital coding metamaterials, programmable metamaterials, and information metamaterials, and realized their first demonstrations. Dr. Cui received the National Natural Science Awards of China in 2014 and 2018, respectively. Based on Clarivate Analytics, he was a Highly Cited Researcher (Web of Science) in 2019 and 2020, and his researches have been widely reported by Nature News, Science, MIT Technology Review, Scientific American, New Scientists, etc. Dr. Cui is an IEEE Fellow.
E-mail: tjcui@seu.edu.cn